Frictional properties of bidisperse granular matter: Effect of mixing ratio

Jean-Christophe Géminard, W Losert

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J.-C. Géminard[1] and W. Losert[2,*]
[1]Laboratoire de Physique de l'Ecole Normale Supérieure de Lyon, 46 Allée d'Italie, 69364 Lyon Cedex
[2]Physics Department, Haverford College, Haverford, Pennsylvania 19041
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The frictional response of granular binary mixtures to an applied shear stress is studied experimentally by sliding a rough plate across a granular surface. The static friction force is found to be up to 25% larger than a linear interpolation between the frictional properties of each component. The dynamical friction coefficient can exhibit a maximum, a minimum, or an oscillatory behavior as a function of mixing ratio, depending on the size ratio or shape of the two components. In addition, visualization of the granular flow makes it possible to show that the shear layer thickness and the characteristic shear displacement, over which a steady state dilation is reached, change linearly with the mass concentration.

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The response of any material to an applied shear stress is an important mechanical property. For granular materials, a sufficiently large shear stress can qualitatively change the material properties from a stationary state to a flowing state for particles in a region close to the shear plane. Nasuno and co-workers [1,2] determined the shear forces of dry granular matter under low applied normal stress in a friction experiment by pushing a plate across the surface of a dry granular medium. This experimental setup has also been used to investigate in detail the frictional properties of a wet granular layer [3], and to study gradual changes in the frictional properties of a granular layer at rest [4] (e.g., waiting time strengthening).

The concept of this experiment is simple: When a plate is moved across the top surface of a flat granular layer, it is submitted to a friction force F. The plate drags the grains that are in contact with its lower surface leading to the fluidization and dilation of a few layers of grains.

The most interesting, and probably most surprising, feature is that, even though particles clearly flow within a shear band, the friction force F is solidlike: F is found to be proportional to the normal applied stress, and independent of the surface area of the sliding plate. It also does not depend significantly on the shear rate. These results hold true for both dry granular matter and materials immersed in water.

However, these studies were restricted to only slightly polydisperse particles with one characteristic particle radius R. That provides one characteristic length scale, which enters into the frictional properties in at least two ways:

(i) When the plate is moved across the top surface of a flat granular layer, the plate drags the grains that are in contact with its lower surface, leading to the motion of particles within several layers of grains. The horizontal velocity profile within the granular layer is roughly exponential, and the characteristic depth D of the shear band is proportional to R (of the order of a few particle diameters).

(ii) As the plate starts to move under an applied shear force, the granular material underneath the plate dilates. Steady state dilation is reached over a horizontal displacement L of the plate, which is about one bead radius R. The total dilation of the layer (vertical displacement of the sliding plate) is only a fraction of one bead radius.

This restriction to materials with only one characteristic length scale leaves many questions about the frictional properties of mixtures that are often technologically relevant. Binary mixtures of beads having two different radii provide a simple example of systems exhibiting two different underlying characteristic sizes.

In this paper we will address the following questions experimentally: *What is the friction coefficient of a binary granular mixture as a function of mixture composition? What are the characteristic lengths L and D in the case of a binary mixture of grains having different sizes?*

Systematic measurements of the frictional properties of binary mixtures as functions of the mixture ratio have not yet been performed to our knowledge [6]. Measurements are challenging because granular mixtures forced to flow usually tend to segregate [7]. One study of the angle of repose, which is related to the frictional property of the granular material, was carried out by Hill *et al.* [8]. They looked experimentally and in simulations at the angle of repose of a binary mixture of granular material, partially filling a long horizontal rotating cylindrical drum. Under many conditions the material can segregate into alternating bands along the axis of the drum. The variation of the angle of repose as a function of the mixing ratio between the two components was found to crucially influence mixing and segregation.

Here, we avoid segregation by studying shear of binary mixtures of particles over very short distances—distances too short to lead to measureable segregation. Due to length limitations of the current apparatus, the shear distance cannot be increased to determine, for which mixture segregation would occur in steady state. For this possibly unstable mixed state of binary mixtures, we report experimental measurements of the yield shear force, dilation, and particle motion within the shear band.

The experimental setup shown in Fig. 1 is similar to that described previously in Refs. [1,2], including the modifications for experiments underwater [3,4]. We measure the friction force necessary to drag a sliding plate on top of a flat

*Present address: Dept. of Physics and IPST, University of Maryland, College Park, MD 20742.

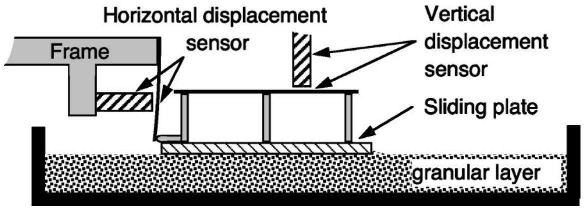

FIG. 1. Experimental setup.

granular layer. The sample consists of a thin layer of granular material (4 mm thick, at least 10 bead diameters [5]) spread in a (5.3×18.5)-cm^2 tray. A transparent acrylic plate $(5.28\times8.15\text{cm}^2)$ of mass $m=24.5$ g is placed on top of the granular material. Good contact between plate and layer is ensured by etching grooves or by gluing a layer of grains on the lower surface of the plate. The plate is pushed across the granular material by a spring (elastic constant $k=189.5$ N/m). The spring and the plate are at contact through a small rounded tip attached to the plate; this allows vertical motion of the plate and ensures that no significant torque is applied. The spring fulcrum is moved toward the plate at constant speed v by a microstep stepper motor (v ranges between 0.1 μm/s and 1 mm/s). Bending of the spring is measured with an inductive displacement sensor (Electro Corporation EMD1050), which indicates the force applied by the spring to the plate with a relative precision better than 0.1%. The vertical position of the plate is measured with a second displacement sensor having a resolution of 0.1 μm. The tray is transparent and allows imaging of the granular material from the side with a fast camera (Kodak Motioncorder SR-Ultra).

The *pure* granular materials used in the experiments consist of slightly polydisperse samples of spherical glass beads (Jaygo, Inc.) of four different sizes (roughly 30, 100, 200, and 450 μm in diameter) and of sand [rough silicate particles, mean size 120 μm (Estes, Inc.)]. The size distribution in each of the samples is roughly Gaussian (Fig. 2) [9]. The mean diameter and the size dispersion are given in Table I.

The samples are prepared by mixing a total amount of 70 g of two granular materials with different mass concentrations. Homogeneous samples are obtained by pouring repetitively the mixture from one cup to another as well as by stirring the content of the cup with a metallic spoon. (The cups are made of cardboard which avoids charging of the grains.) The content of the cup is then poured into the tray. A flat layer is obtained by removing the excess of grains with

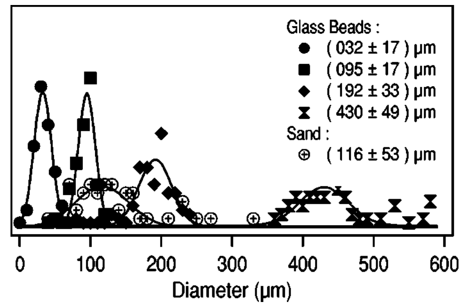

FIG. 2. Size distribution of the grains in the *pure* samples.

TABLE I. Diameter of the grains, size dispersion, and static and dynamic friction coefficients μ_s and μ_d for the *pure* samples. We define the size dispersion as the width of the Gaussian distribution divided by the mean diameter.

Type	Diameter (μm)	Dispersion	μ_s	μ_d
Glass beads	32 ± 17	0.51	0.84	0.58
	95 ± 17	0.18	0.70	0.59
	192 ± 33	0.17	0.52	0.50
	430 ± 49	0.11	0.51	0.47
Sand	116 ± 53	0.46	0.80	0.76

the help of a vertical plate sliding along the edges of the side walls. The leveled granular layer that remains after this last operation is not submitted to any shear and, thus, to any segregation during preparation.

The friction force F is measured while sliding the plate in the *stick-slip* regime (Fig. 3). The static friction force F_s is obtained by measuring the maximum spring displacement d_{max} before a slip event. An estimate of the dynamic friction force F_d is extracted from the same experimental results. Indeed, assuming that F_d is constant during a slip event, one can show that the value of F_d corresponds to the mean displacement d_{mean} of the spring. Because *stick-slip* motion is the pattern generally observed, the experimental setup does not allow to measure the dependance of the dynamic friction coefficient on the driving velocity v. We limit our study to the static $(v=0)$ and dynamic $(v\neq0)$ values of the friction coefficients assuming that μ_d does not depend on the shear rate. (In fact, μ_d exhibits some hysteresis [2] but we assume, as a first approximation, that μ_d is constant and study the dependance of the *so-defined* dynamic friction coefficient on the mixture composition.) The typical velocity, during a slip event, is about 1 cm/s. The experimental results do not exhibit any sensitivity to the driving velocity v, and we choose, for convenience, to give results for $v=11.3$ μm/s. The friction coefficients μ and displacements d are simply related through: $\mu=kd/mg$ where k is the spring constant, m the mass of the sliding plate, and g the gravity constant. We perform most of the experiments with the same sliding plate ($m=24.5$ g with grooves); We deduce the friction coefficient from the measured force that is proportional to the plate weight [10]. The uncertainty in the measurements is better

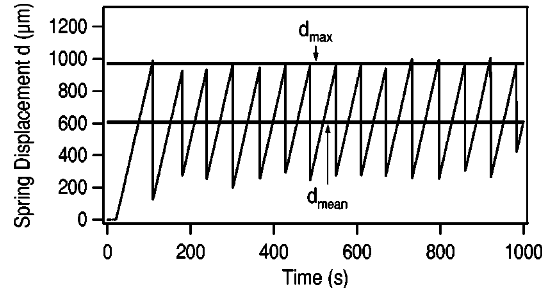

FIG. 3. Spring displacement as a function of time at a constant stage speed of $v=11.3$ μm/s. (The sample is a mixture of 200 μm-060 μm glass beads at 50% in mass concentration.)

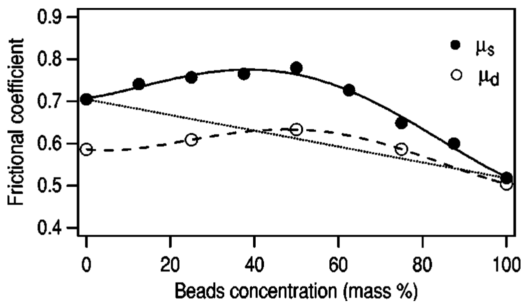

FIG. 4. Friction coefficients as functions of the mass concentration (200-μm beads in 100-μm beads).

than 2%. (The most important part of the measurement uncertainty originates in the layer preparation.)

The values of the friction coefficients measured for the *pure* samples using the plate with grooves are reported in Table I. We observe that the measured static friction coefficient is larger for the smaller glass beads than for bigger ones, and that the static friction coefficient is larger for sand than for glass beads. In addition, the size polydispersity of our samples of small beads is significantly larger than that of larger beads (Table I) and their shape is less well defined. These observations are in agreement with measurements of angles of avalanche performed by Felix *et al.* in rotating drums [11]. The results of that experimental study demonstrate that the polydispersity and the roughness of the grains lead to an increase in the static friction coefficient.

The typical behavior of the static and dynamic friction coefficients μ_s and μ_d as functions of the mixture composition is shown in Fig. 4. Segregation is expected to occur when the granular layer is sheared. Nevertheless, we do not measure any systematic drift of the friction force during the experimental time, even if the sliding plate is typically pushed across 1 cm of the sample free surface. Thus, if segregation occurs while the measurements are performed, its effects on the force measurements are not significant. (Moreover, imaging of the sample from the side does not clearly highlight any segregation. If segregation occurs, it does not lead to a total separation of the components.) The quantitative results depend within a constant multiplicative

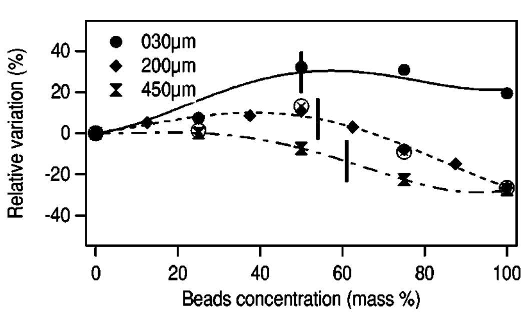

FIG. 5. Relative variation of the static friction coefficient as a function of the concentration (relative to 100-μm beads). 30-, 200-, and 450-μm beads are added to 100-μm beads. The crosses in open circles correspond to the case of 200-μm beads with sand glued at the lower surface of the plate. Vertical dashes indicate the concentration at which the mixture reaches its maximum density.

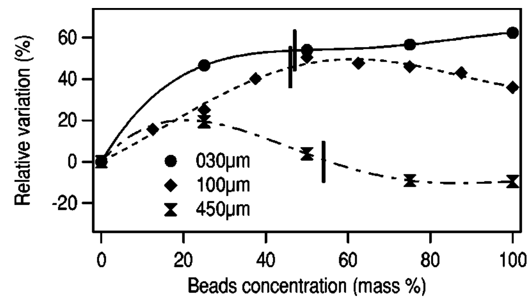

FIG. 6. Relative variation of the static friction coefficient as a function of the concentration (relative to 200-μm beads). 30-, 100-, and 450-μm beads are added to 200-μm beads. Vertical dashes indicate the concentration at which the mixture reaches its maximum density.

factor on the contact between the sliding plate and the granular layer. The measured friction force is larger when sand is glued to the lower surface of the plate than it is when grooves are etched. Direct observations of particle velocities from the side indicate that noticeable slip occurs at the boundary with grooves, while the slip velocity is close to zero with glued sand. Nevertheless, the experimental results are qualitatively the same with both surface treatments, and the relative variations of the friction coefficient as functions of the concentration agree quantitatively [amplitude and position of the maximum (Fig. 5)]. Thus, we conclude that the measured variations of the friction force do not sensitively depend on the geometry of the driving wall, but are due to intrinsic variations of the frictional properties of the material.

The static friction coefficient exhibits, at intermediate concentration, a maximum which is always above the linear interpolation between the values obtained for the *pure* components. (The amplitude of maximum can be up to 25% above the linear interpolation.) These features hold true for all the glass beads mixtures we tested experimentally (30-, 200-, and 450-μm beads in 100-μm beads; 30-, 100-, and 450-μm beads in 200-μm beads; and 100-, 200-, and 450-μm beads in sand). On the other hand, the position and amplitude of the maximum value depend on the nature of the two *pure* components (Figs. 5, 6, and 7). We nevertheless do not find any simple relation between the position and amplitude of the maximum and the size ratio of the grains in the case of samples of spherical glass beads.

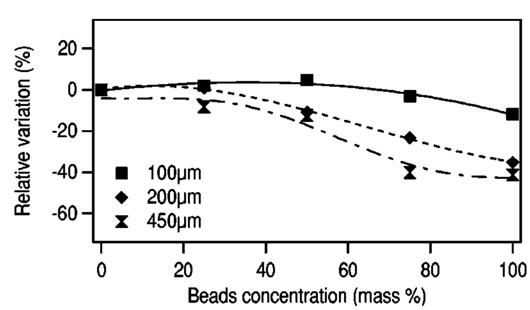

FIG. 7. Relative variation of the static friction coefficient as a function of the concentration (relative to sand). 100-, 200-, and 450-μm beads are added to 120-μm sand grains.

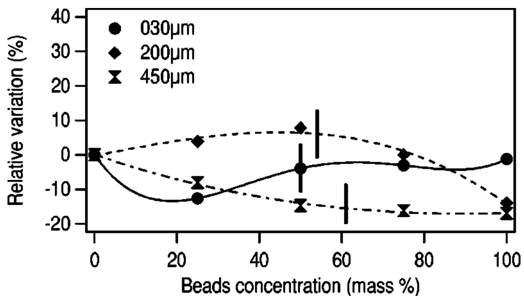

FIG. 8. Relative variation of the dynamic friction coefficient as a function of the concentration (relative to 100-μm beads). 30-, 200-, and 450-μm beads are added to 100-μm beads. Vertical dashes indicate the concentration at which the mixture reaches its maximum density.

The dynamic friction coefficient μ_d exhibits a less systematic behavior (Figs. 8, 9, and 10). The size ratio of the glass beads that form the mixture is not enough to predict whether the friction coefficient will exhibit a maximum or a minimum. We can nevertheless point out that the relative variation of μ_d is systematically smaller than the variation of μ_s.

In addition, we carried out measurements of the characteristic length of dilation L and of the depth D of the shear band.

When subjected to shear deformation, the granular layer dilates. When the plate, initially at rest, is pushed by the spring, it rises up over a characteristic horizontal length L. The experimental setup makes it possible to measure L by performing experiments on a granular layer immersed in water. The method is described in an earlier publication [3]. In the case of monodisperse spherical glass beads, the characteristic length L equals the bead radius R. In granular mixtures, it, therefore, provides an indication of a characteristic size scale of the material. The measurements of L, given in Fig. 11, demonstrate that the underlying characteristic size L changes from one radius to the other roughly linearly with the mass concentration.

Additionally, imaging the individual motion of the grains from the side of the sample makes it possible to describe the granular flow within the sample during the slip events. The velocity profile within the depth of the sample is roughly

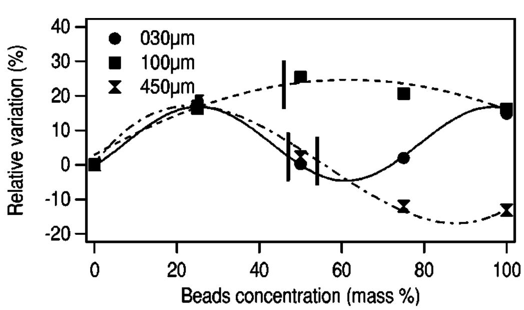

FIG. 9. Relative variation of the dynamic friction coefficient as a function of the concentration (relative to 200-μm beads). 30-, 100-, and 450-μm beads are added to 200-μm beads. Vertical dashes indicate the concentration at which the mixture reaches its maximum density.

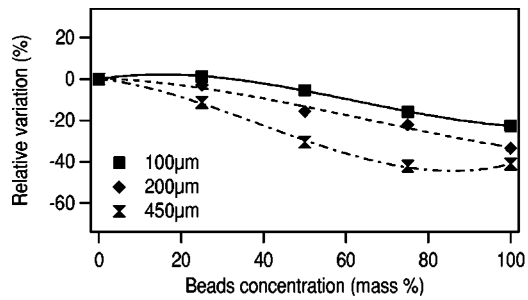

FIG. 10. Relative variation of the dynamic friction coefficient as a function of the concentration (relative to sand). 100-, 200-, and 450-μm beads are added to 120-μm sand grains.

exponential with a characteristic length D which does not depend on the plate velocity. The characteristic depth D depends linearly on the mass the concentration (Fig. 12), and the granular flow occurs within two to three granular *layers* defined by L.

In summary, we studied experimentally frictional properties of binary mixtures as functions of the mixture ratio.

(i) The static friction force exhibits a maximum at intermediate mixture ratio (the static friction force can be up to 25% larger than a linear interpolation between the frictional properties of each component). This feature does not change with size ratio, nature and/or shape of the grains.

(ii) The dynamic friction force shows a less systematic behavior; it can decrease or increase relative to a linear interpolation depending on the mixture components. Generally, the relative variation of the dynamic friction force is less than the one measured for the static friction force.

(iii) The shear layer thickness and the characteristic shear displacement over which a steady state dilation is reached change roughly linearly with the mass concentration.

The variation of the friction force as a function of the mixture ratio is not yet understood. At least two factors may contribute to the variation of the friction force for mixtures:

(1) The packing density of granular mixtures changes with mixture ratio (Table II). However, we measured that the extremum of the friction force generally does not correspond to the extremum densities measured in addition (Figs. 5, 6, 8, and 9).

(2) The number of beads in the shear layer depends strongly on the mixture composition. Many experiments on granular shear at small pressures have found that particles

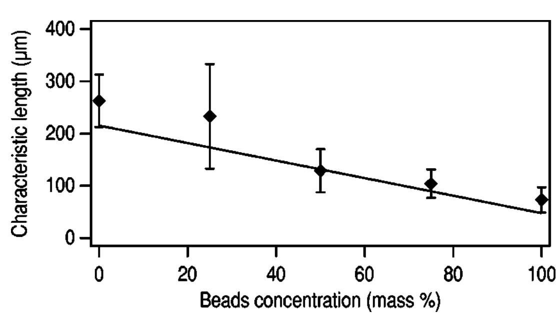

FIG. 11. Characteristic length of the mixture as a function of the concentration (100 μm in 450-μm glass beads). The solid line connects the radii of the two components.

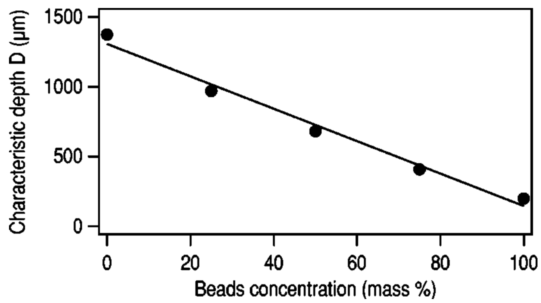

FIG. 12. Characteristic depth of the velocity profile as a function of the concentration (mixture of 100-μm and 600-μm spherical colored glass beads).

TABLE II. Position and value of the maximum density for the mixtures of glass beads. The values of the density of the *pure* components are given on the diagonal. The percentages correspond to the concentration of beads the radii of which are given on the left-hand side, in the beads the radii of which are given on top. The maximum densities are given in the same frame.

	30 μm	100 μm	200 μm	500 μm
30 μm	0.73	50%, 0.76	47%, 0.80	59%, 0.85
100 μm	50%, 0.76	0.72	46%, 0.75	39%, 0.82
200 μm	53%, 0.80	54%,0.75	0.71	46%,0.75
500 μm	41%, 0.85	61%, 0.82	54%, 0.75	0.70

move within a shear band with a characteristic depth D of about a few particle diameters. The result holds true in binary mixtures, provided that the characteristic size of the underlying particles is taken to be the mean radius L by weight. The variation of the number of contacts between grains in the shear band could account for the observed variation of the friction force.

The simple dependance of the characteristic lengths D and L on the mixture composition is still not understood. The adaptation of new hydrodynamical approaches of the problem of granular flows to the the case of binary mixtures could help understanding our experimental results [12]. Moreover, the experimental results show that the frictional properties of mixtures cannot simply be extrapolated from the frictional properties of the individual components. For a given mixture, the static friction force can exhibit a local maximum at intermediate mixing ratio, while the dynamic friction force may exhibit a minimum or a maximum. The frictional properties included in any model of mixing and segregation never approach this complexity to our knowledge [8]. The introduction of maxima and minima in the friction forces into models of granular mixtures, together with further experimental measurements on unstable mixed or segregated states, may provide important clues to the puzzling and often seemingly unpredictable behavior of granular mixtures.

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[1] S. Nasuno, A. Kudrolli, and J.P. Gollub, Phys. Rev. Lett. **79**, 949 (1997).
[2] S. Nasuno, A. Kudrolli, A. Bak, and J.P. Gollub Phys. Rev. E **58**, 2161 (1998).
[3] J.-C. Geminard, W. Losert, and J.P. Gollub, Phys. Rev. E **59**, 5881 (1999).
[4] W. Losert, J.-C. Geminard, S. Nasuno, and J.P. Gollub, Phys. Rev. E **61**, 4060 (2000).
[5] Direct observation of the motion of the grains from the side shows that the granular flow never extends up to the bottom of the tray, and that the grains belonging to the bottom layers do not move. We also checked experimentally that, in this limit, the measured values of the frictional coefficients do not depend on the thickness of the granular layer.
[6] Abou-Chakra and Tüzün reported a study of the friction coefficient of a smooth wall in contact with a monolayer of a granular mixture. Such an experiment is very similar to solid on solid friction, but differs from friction of a deep granular bed in an important aspect: there is no flow within the granular layer, i.e., a two-dimensional network of particle contacts breaks under excess shear forces, while a three-dimensional network breaks in a surface region of a deep granular bed. H. Abou-Chakra, U. Tüzün, Chem. Eng. Sci. **54**, 5901 (1999); **54**, 5913 (1999).
[7] T. Shinbrot and F.J. Muzzio, Phys. Today **53**(3), 25 (2000).
[8] K. M. Hill, J. Kakalios, K. Yamane, Y. Tsuji, and A. Caprihan, in *Conference Proceedings of Powders and Grains 97*, edited by J. T. Jenkins (Balkema, Rotterdam, 1997), pp. 483–486.
[9] The size distribution is obtained by observing the grains in the microscope. The diameters of a given number of grains (typically 500) are then measured with the help of an image analysis software (NIH Image).
[10] The proportionality between the friction force and the normal stress has been clearly demonstrated in the case of monodisperse granular material. The most important point for our study is that the friction force depends on the mixture concentration, the normal stress remaining constant.
[11] G. Felix, U. D'Ortona, and V. Falk, in *Powders and Grains 2001*, edited by Kishino (Swets & Zeitlinger, Lisse, 2001), pp. 475–478.
[12] W. Losert, L. Bocquet, T.C. Lubensky, and J.P. Gollub, Phys. Rev. Lett. **85**, 1428 (2000).